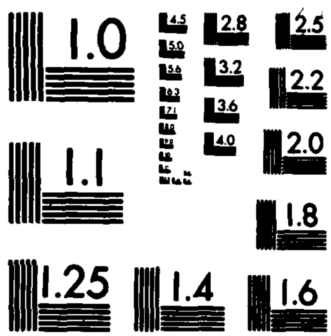
AD-A127 783 CHARACTERIZATIONS OF MULTIVARIATE CLASSES OF LIFE DISTRIBUTIONS(U) ILLINOIS UNIV AT CHICAGO CIRCLE DEPT OF MATHEMATICS E EL-NEWEIHI NOV 82 AFOSR-TR-83-0342 1/1

UNCLASSIFIED AFOSR-80-0170 F/G 12/1 NL



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Characterizations of Multivariate Classes of Life Distributions

by

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November, 1982

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Research sponsored by the Air Force Office of Scientific Research, AFSC, USAF, under Grant AFOSR 80-0170.

Key words and Phrases: Increasing failure rate average, new better than used, increasing sets, characterizations.

AMS 1980 subject classifications: Primary 62H05, secondary 62N05

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 83-0342	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) CHARACTERIZATIONS OF MULTIVARIATE CLASSES OF LIFE DISTRIBUTIONS		5. TYPE OF REPORT & PERIOD COVERED TECHNICAL
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Emad El-Neweihi		8. CONTRACT OR GRANT NUMBER(s) AFOSR-80-0170
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Mathematics, Statistics, and Computer Science, University of Illinois at Chicago, P.O. Box 4348, Chicago IL 60680		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS PE61102F; 2304/A5
11. CONTROLLING OFFICE NAME AND ADDRESS Mathematical & Information Sciences Directorate Air Force Office of Scientific Research Bolling AFB DC 20332		12. REPORT DATE NOV 82
		13. NUMBER OF PAGES 5
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION ST. MENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY TES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Increasing failure rate average; new better than used; increasing sets; characterizations.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Let $I(N)$ be the class of nonnegative random vectors T for which $\min_{1 < i < n} a_i T_i$ is IFRA (NBU) for all $a_i > 0$, $i=1,\ldots,n$, where n is an arbitrary positive integer. Characterizations of the classes I and N which are useful in deriving some of their properties are obtained.

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Abstract

Let $\mathcal{I}(\mathcal{N})$ be the class of nonnegative random vectors $\underline{T}$ for which $\min_{1 \leq i \leq n} a_i T_i$ is IFRA (NBU) for all $a_i > 0$, $i=1,\ldots,n$, where n is an arbitrary positive integer. Characterizations of the classes $\mathcal{I}$ and $\mathcal{N}$ which are useful in deriving some of their properties are obtained.

1. Introduction and Preliminaries.

Various multivariate extensions of the univariate classes of increasing failure rate average (IFRA) and new better than used (NBU) distributions are now available (see for example Block and Savits (1980), El-Neweihi (1981) and Marshall and Shaked (1982)). In this paper we characterize two of those classes and utilize the characterizations to derive simple proofs of some properties of the two classes.

First let us present the notation, terminology and definitions used throughout this paper. Let $\underline{x}$ and $\underline{y}$ be two vectors in R^n, then $\underline{x} \leq \underline{y}$ ($\underline{x} < \underline{y}$) means $x_i \leq y_i$ ($x_i < y_i$), $i=1,\ldots,n$. Given a vector $\underline{x} \in R^n$, $Q_{\underline{x}} = \{\underline{y}:\underline{x} < \underline{y}\}$. The set $\{\underline{x}:\underline{0} \leq \underline{x}\}$ is denoted by R_+^n, where $\underline{0}=(0,\ldots,0)$. A function $f:R_+^n \rightarrow R_+$ is said to be nondecreasing if $\underline{x} \leq \underline{y}$ implies that $f(\underline{x}) \leq f(\underline{y})$. A subset $U \subseteq R^n$ is said to be an increasing set if $\underline{x} \in U$ and $\underline{x} \leq \underline{y}$ imply that $\underline{y} \in U$.

Throughout the remainder of this paper all the random vectors (random variables) considered are assumed to be nonnegative. Also all the random variables considered are not degenerate at 0.

2. Characterizations and Properties.

In this section we characterize the following two classes:

$$I = \{\underline{T}=(T_1,\ldots,T_n): \min_{1\leq i\leq n} a_iT_i \text{ is IFRA for all } a_i>0, i=1,\ldots,n$$

where n is a positive integer},

$$N = \{\underline{T}=(T_1,\ldots,T_n): \min_{1\leq i\leq n} a_iT_i \text{ is NBU for all } a_i>0, i=1\ldots,n$$

where n is a positve integer}.

Recall that a random variable T is IFRA if $P(T > \alpha t) \le \{P(T>t)\}^{\alpha}$ for every $t \ge 0$ and every $0 < \alpha < 1$. Also a random variable T is NBU if $P(T > t) \le P(T > \alpha t)P(T>(1-\alpha)t)$ for every $t \ge 0$ and every $0 < \alpha < 1$. Well known characterizations for these univariate classes of lifetimes have been obtained by Block and Savits (1976) and Marshall and Shaked (1982). Characterizations of similar type for the classes I and N are now derived. First we need the following lemma.

Lemma 2.1. Let $\underline{T}=(T_1,\ldots,T_n)$ be a random vector and f_1, f_2 be two nonnegative functions defined on R_+^n. Assume that

$$Ef_i(\underline{T}) \le \{Ef_i(\underline{T}/\alpha)\}\{Ef_i^{1-\alpha}(\underline{T}/1-\alpha)\}, \tag{2.1}$$

for i=1,2 and $0 < \alpha < 1$. Then (2.1) is valid with f_1+f_2 replacing f_i.

Proof. In the following the second inequality follows from Hölder inequality and the third inequality follows from Minkowski inequality for the L_α-norm, $0 < \alpha < 1$:

$$\begin{aligned} E(f_1+f_2)(\underline{T}) &= Ef_1(\underline{T}) + Ef_2(\underline{T}) \\ &\le \{Ef_1^{\alpha}(\underline{T}/\alpha)\}\{Ef_1^{1-\alpha}(\underline{T}/1-\alpha)\} + \{Ef_2^{\alpha}(\underline{T}/\alpha)\}\{Ef_2^{1-\alpha}(\underline{T}/1-\alpha)\} \\ &\le \{\sum_{i=1}^{2} (Ef_i^{\alpha}(\underline{T}/\alpha))^{1/\alpha}\}^{\alpha}\{\sum_{i=1}^{2} (Ef_i^{1-\alpha}(\underline{T}/1-\alpha))^{1/1-\alpha}\}^{1-\alpha} \\ &\le \{E(f_1+f_2)^{\alpha}(\underline{T}/\alpha)\}\{E(f_1+f_2)^{1-\alpha}(\underline{T}/1-\alpha)\}. \end{aligned}$$

In what follows let C denotes the set $\{f: f(\underline{x}) = \prod_{i=1}^{n} f_i(x_i)$, where $f_i: R_+ \to R_+$ is nondecreasing, i=1,...,n and n is a positive integer$\}$. We are now ready to prove the main result of this paper.

Theorem 2.2. Let $\underline{T} = (T_1,\dots,T_n)$ be a random vector. Then

(i) $\underline{T}\in I$ if and only if

$$Ef(\underline{T}) \le \{Ef^{\alpha}(\underline{T}/\alpha)\}^{1/\alpha}, \tag{2.2}$$

for every $f\in C$ and every $0 < \alpha < 1$.

(ii) $\underline{T}\in N$ if and only if

$$Ef(\underline{T}) \le \{Ef^{\alpha}(\underline{T}/\alpha)\}\{Ef^{1-\alpha}(\underline{T}/1-\alpha)\}, \tag{2.3}$$

for every $f\in C$ and every $0 < \alpha < 1$.

Proof.(i) First assume that $T\in I$. Then Clearly (2.2) is valid for every function f of the form $f(\underline{x}) = \prod_{i=1}^{n} I_{B_i}(x_i)$, where B_i is an increasing subset of $(0,\infty)$, $i=1,\dots,n$. As in Block and Savits (1976), the L_α-norm inequality for $0 < \alpha < 1$ extends the validity of (2.2) to finite nonnegative linear combinations of functions of this form. The monotone convergence theorem now extends the validity of (2.2) to any function $f\in C$.

Conversely if (2.2) is true, then $\underline{T}\in I$ follows by taking $f(\underline{x}) = \prod_{i=1}^{n} I_{(t/a_i,\infty)}(x_i)$, where $t \ge 0$ and $a_i > 0$, $i=1,\dots,n$.

(ii) The proof follows readily by lemma 2.1 and by using similar steps to the ones used in the proof of (i). The details are therefore omitted.

Remark 2.3. Let C_1 be the class of all nonnegative, nondecreasing and Borel measurable functions which are defined on R_+^n. Block and Savits (1980) defined a multivariate class of life distributions by requiring that (2.2) is true for all $f\in C_1$. Also Marsall and Shaked (1982) defined a multivariate class of NBU distribution which is characterized by the validity of (2.3) for all $f \in C_1$. Obviously $C_1 \supset C$.

The characterizations obtained in theorem 2.2 are useful in deriving simple proofs for some desirable properties for both I and N. We illustrate this by the following corollary.

Corollary 2.4. Let $\underline{T}=(T_1,\ldots,T_n)$ be a random vector then

(i) $\underline{T}\varepsilon I$ ($\underline{T}\varepsilon N$) implies that $(T_{i_1},\ldots,T_{i_k})\varepsilon I$ ($(T_{i_1},\ldots,T_{i_k})\ \varepsilon N$) for every $1 \le i_1<\ldots<i_k \le n$ and every $1 \le k \le n$.

(ii) Let $g_i:R_+ \to R_+$ be a nondecreasing function such that

$g_i(x/\alpha) \le g_i(x)/\alpha$ for every $x\varepsilon R_+$ and every $0 < \alpha < 1$, $i=1,\ldots,n$. Then $(g_1(T_1),\ldots,g_n(T_n))\varepsilon I$ ($(g_1(T_1),\ldots,g_n(T_n))\varepsilon N$) whenever $\underline{T}\varepsilon I$ ($\underline{T}\varepsilon N$).

Proof. In view of (2.2) and (2.3) the proofs are clear and are left to the reader.

Remark 2.5. It should be noted that without (2.2) and (2.3) the proofs required to establish the above corollary are less straightforward.

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